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Ion-Focusing Properties of a Three-Element Quadrupole Lens System*

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The focusing properties of a three-element quadrupole lens system have been studied, and the results of thick-lens calculations are presented in the form of graphs showing the field-strength parameters and magnifications, as functions of object and image distances.

I. INTRODUCTION

In a previous paper[1] the ion-focusing properties of a quadrupole lens pair were analyzed. The results were presented in the form of graphs giving lens strengths and magnification factors versus object and image distances. The present paper gives a similar analysis for a three-element lens system in which the middle section has twice the length of each of the outside sections and the two outside sections have equal field-strength parameters.

II. CALCULATION OF LENS STRENGTHS

Figure 1 shows the particle trajectories in two planes of the three-element lens. The $x-y$ plane is herein referred to as the diverging plane (outside lenses diverging); the $x-z$ plane is referred to as the converging plane. In the case shown, the object and image distances are the same in both planes (stigmatic focusing). The following graphical procedure of solving the equations for the lens strengths can be used also if astigmatic focusing is wanted.

The procedure is as follows: The relationship between the lens strengths and the object and image distances of single sections is computed and presented in graphical form. These graphs give the equivalent of the optical lens equation

$$1/a+1/b=1/f,$$

except that the object and image distances are measured from the boundary of the lens rather than from the principal planes. Given the focal properties of single sections, a ray can be traced through the complete system. By trial and error, the lens strengths can then be adjusted to give the desired object and image distances for the whole system. The procedure is analogous to the matching of logarithmic derivatives at boundaries between regions of constant potential in wave mechanical problems.

Figure 2 shows the trajectories in both planes of a single section of length L. Equations relating the object and image distances and the field-strength parameter (wave number) k can be found by applying proper boundary conditions to Eq. (1) in reference 1. The results are

Trigonometric (or converging) plane

$$L/b_t=-kL\cot(kL+\phi_t) \tag{1}$$

$$L/a_t=kL\cot\phi_t \tag{2}$$

Hyperbolic (or diverging) plane

$$L/b_h=-kL\begin{matrix}\coth\\ \tanh\end{matrix}(kL+\phi_h) \tag{3}$$

$$L/a_h=kL\begin{matrix}\coth\\ \tanh\end{matrix}\phi_h. \tag{4}$$

In Eqs. (3) and (4), the coth function is used when the product a_hk is smaller than 1, and the tanh function is used when a_hk is larger than 1.

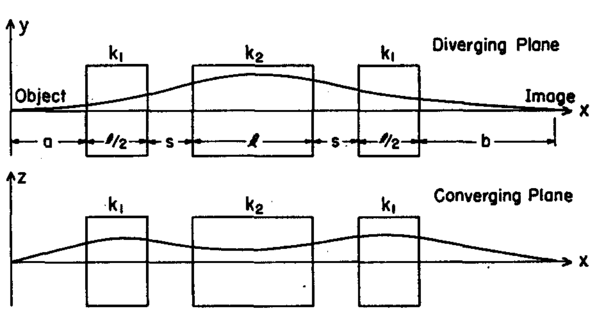

Fig. 1. Particle trajectories in the two planes of a three-element quadrupole lens system.

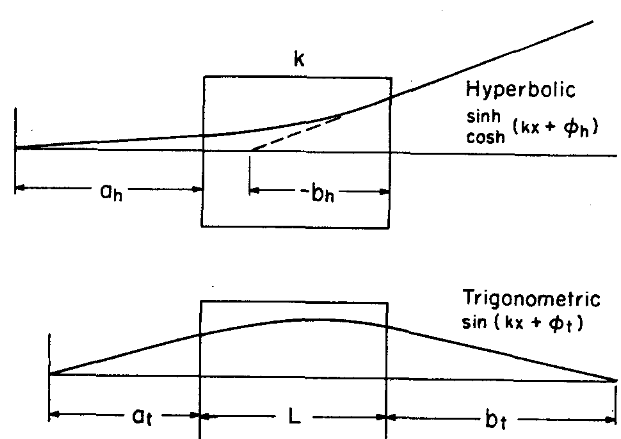

Fig. 2. Trajectories in two planes of a single lens section.

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[1] H. A. Enge, Rev. Sci. Instr. **30**, 248 (1959).

The field-strength parameter, or wave number k, is given by the relationships

$$k=\frac{1}{d}\left(\frac{3.2\pi IN}{B\rho}\right)^{\frac{1}{2}}\text{cm}^{-1}\text{ magnetic lens,}$$

$$k=\frac{2}{d}\left(\frac{nV}{E}\right)^{\frac{1}{2}}\text{cm}^{-1}\text{ electrostatic lens,}$$

where d is the aperture diameter of the lens in centimeters (pole tip to pole tip). In the magnetic case, $B\rho$ is the magnetic rigidity of the particles in gauss-centimeters, and IN is the number of ampere turns *per pole* available for driving the flux through the air gap. In the electrostatic case, V is the potential on the poles in volts (+ and −), and n is the number of elementary charges carried by the particle. In a nonrelativistic case, E is the kinetic energy of the particle in electron-volts. In general it is $E=p\beta c/2$ (in electron-volts), where p is particle momentum mv, c is the velocity of light, and $\beta=v/c$.

The two sets of equations (1) and (2), and (3) and (4) have been solved numerically, and the results are given in Figs. 3 and 4. In these figures are plotted the inverse of the image distance versus the lens strength and the inverse of the object distance. The quantities are made nondimensional by multiplication or division by L, the length of the section.

For the complete three-element lens system, the notations used are as follows: The field-strength parameter is called k_1 for the outside section and k_2 for the inside section. The lengths are ℓ for the inside section and $\ell/2$ for each of the two outside sections. With the aids of the graphs,

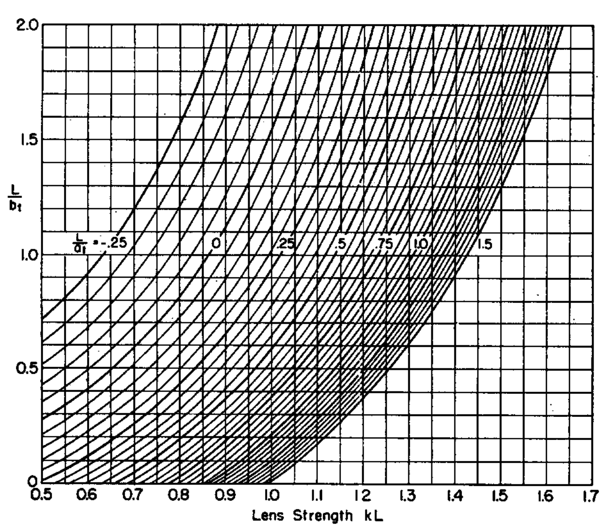


FIG. 3. Inverse image distance L/b_t versus lens strength kL and inverse object distance L/a_t for a trigonometric (converging) lens section (cf. Fig. 2).

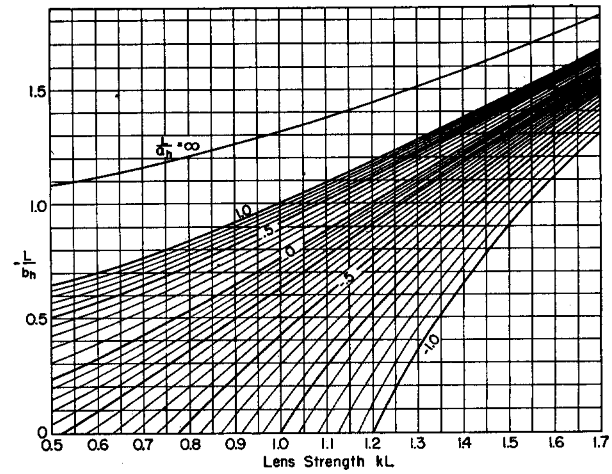
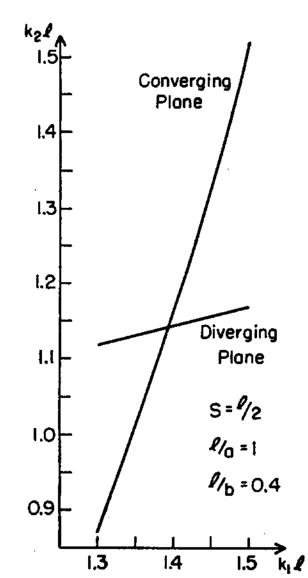


FIG. 4. Negative inverse image distance $-L/b_h$ versus lens strength kL and inverse object distance L/a_h for a hyperbolic (diverging) lens section (cf. Fig. 2).

Figs. 3 and 4, one finds three or four sets of $k_1\ell$ and $k_2\ell$ that will give correct distances a and b for the converging plane. The same is done for the diverging plane. Thereafter, $k_1\ell$ is plotted versus $k_2\ell$ for both cases, as illustrated by an example in Fig. 5. The intersection of the two curves gives a set of $k_1\ell$ and $k_2\ell$ that will give correct focusing in both planes.

Figures 6 and 7 present the final results of the calculations of the lens strengths for a three-element lens system for image and object distances between ℓ and infinity. In Fig. 6, the separation between the lens sections is 0; in Fig. 7, it is equal to the length of the outside section $s=\ell/2$. In the figures are also shown, by dotted lines, the lens-strength parameters for quadrupole lens pairs[1] with the same total lens lengths as the three-element systems (separations $s=0$ and $s=\ell$, respectively).

FIG. 5. Graphical determination of a set of lens strengths $k_1\ell$, and $k_2\ell$ that simultaneously satisfies the focusing conditions in the diverging plane and the converging plane (cf. Fig. 1).



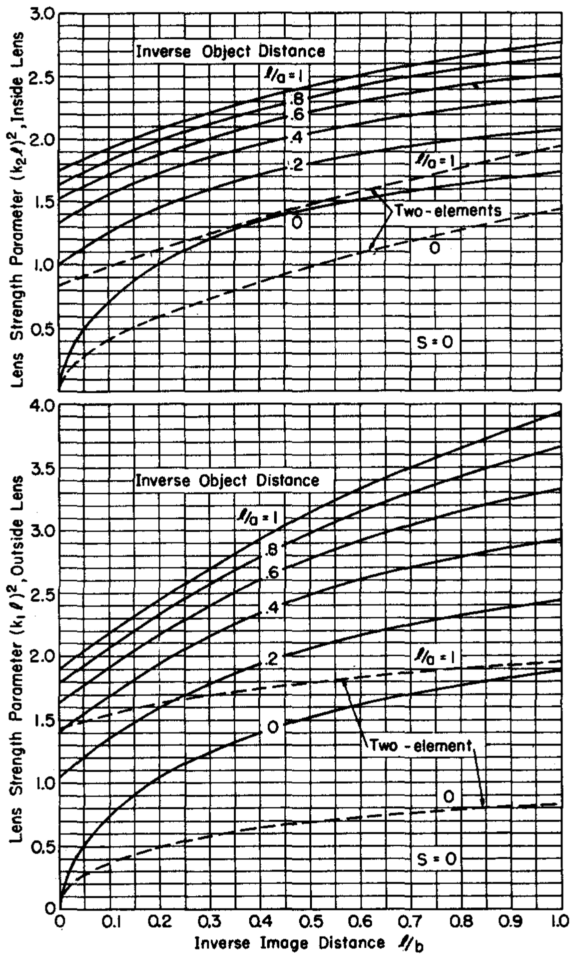


FIG. 6. Lens-strength parameters for three-element lens system versus inverse object and image distances for zero lens separations, $s=0$ (cf. Fig. 1). Two curves for a two-element system (from reference 1) are shown for comparison.

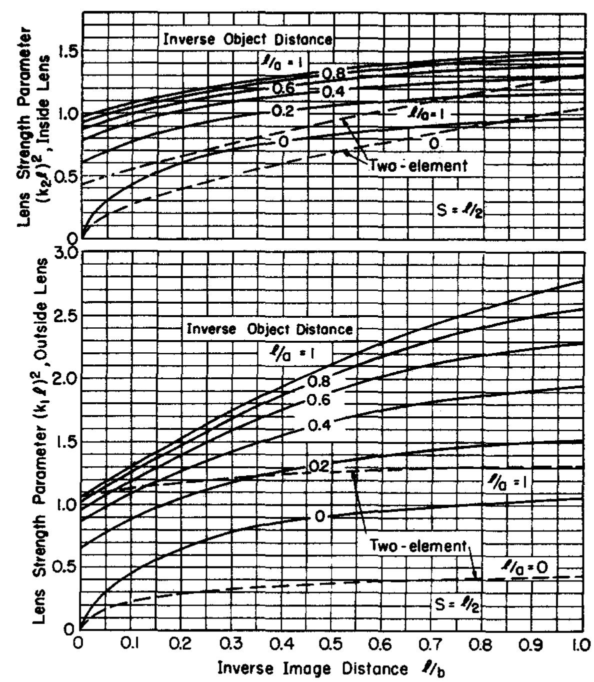


FIG. 7. Lens-strength parameters for three-element lens system versus inverse object and image distances for lens separations $s=\ell/2$ (cf. Fig. 1). Two curves for a two-element system are shown for comparison.

III. MAGNIFICATION

The magnification factors have been computed by use of the familiar formula from optics

$$M=b_1/a_1. \tag{5}$$

Here b_1 and a_1 are image and object distances, respectively, as measured from the principal planes.

For symmetric lens systems, such as those discussed here, the principal points can be found by tracing a ray crossing the axis at the midpoint of the system through the system. The points of intersection between the outside straight-line sections of this ray and the axis are the principal points.

The described method has been used to find the principal points for both planes of lens systems adjusted to stigmatic focus for sets of object and image distances given by ℓ/a and $\ell/b=0.05$, 0.1, 0.2, 0.4, 0.6, 0.8, and 0.10. Figures 3 and 4 were used to trace the ray from the midpoint. The resulting magnification factors calculated

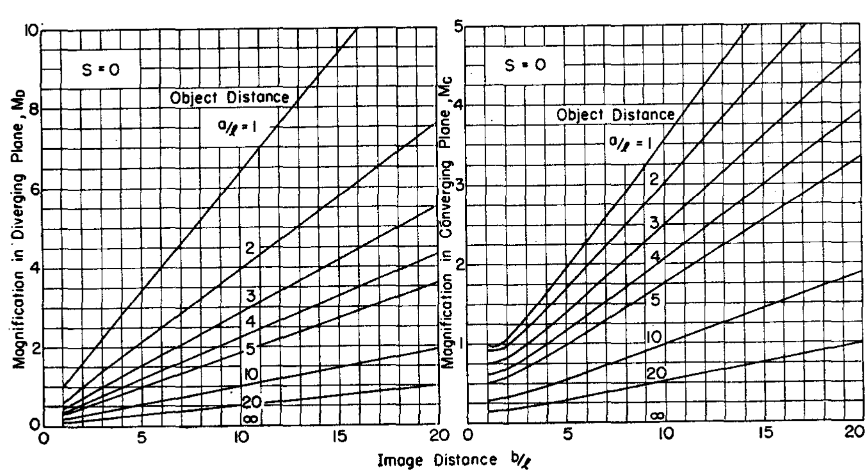


FIG. 8. Magnification factors versus object and image distances for lens separation $s=0$. ("Diverging plane" has outside lenses diverging; "converging plane" has outside lenses converging.)

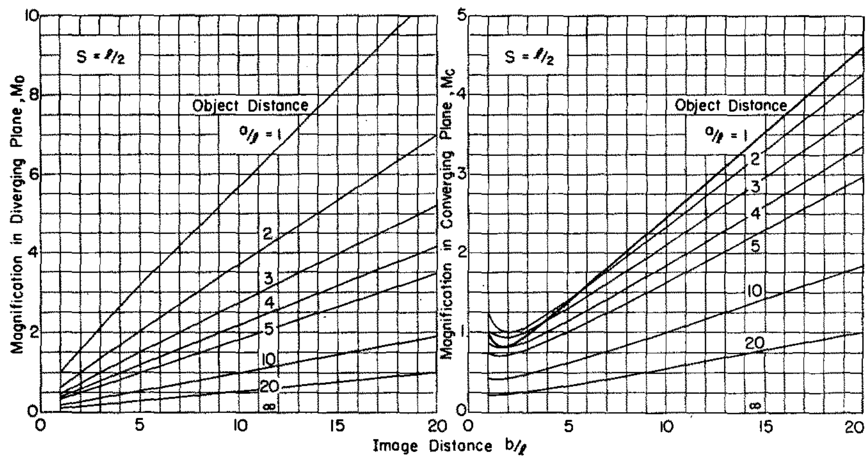


FIG. 9. Magnification factors versus object and image distances for lens separation $s=\ell/2$. ("Diverging plane" has outside lenses diverging; "converging plane" has outside lenses converging.)

from Eq. (5) are plotted in Figs. 8 and 9 for $s=0$ and $s=\ell/2$, respectively.

Since the three-element lens system is symmetric, one might naively expect that the magnification factor should always be larger than unity when the image distance b is larger than the object distance a (both measured from the lens boundaries). This is not always the case in the converging plane (for example, $a=\ell$, $b=2\ell$, Fig. 9). For these cases, the principal points are outside the image points. An optical analogy is provided by two identical thin convex lenses separated by a distance slightly larger than twice the focal length.

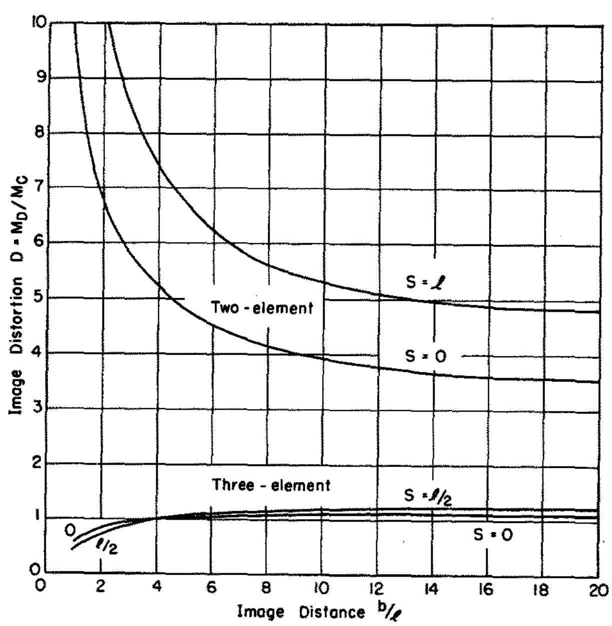


FIG. 10. Image distortion $D=M_D/M_C$ for two-element lens system compared with image distortion of three-element system. The object distance is arbitrarily chosen as $a=4\ell$.

IV. COMPARISON BETWEEN TWO- AND THREE-ELEMENT SYSTEMS

Figures 6 and 7 clearly show that a three-element lens system needs higher lens strength for the same total lens length than a quadrupole pair. This of course is a disadvantage. The advantage of the three-element system is that it has much less distortion than the two-element system. For the purpose of this discussion, distortion is defined as

$$D=M_D/M_C. \quad (6)$$

This factor is the ratio of the long side to the short side of the rectangular image formed when the object is a square. Figure 10 gives distortion factors D versus image distance for an arbitrarily chosen object distance $a=4\ell$, and for the four different lens systems analyzed in this and the previous paper.[1]

Because of the fringing field effect, the length of the pole pieces for an inside lens should be made equal to

$$\ell_{pi}=\ell-fd. \quad (7)$$

The actual length of the pole pieces of the outside sections should be made equal to

$$\ell_{po}=\ell/2-fd. \quad (8)$$

Here d is the aperture diameter, and the factor f depends upon the design. For a particular design studied in great detail by I. E. Dayton *et al.*,[2] the factor was found to be $f=0.57$.

[2] I. E. Dayton, F. C. Shoemaker, and R. F. Mozley, Rev. Sci. Instr. **25**, 485 (1954).