

*Developed by:*

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# CHARGE STATES ANALYSIS CODE

DOCUMENTATION

# About the program

The present program is designed to calculate the expected charge states produced once an ion beam passes through a stripping point and their respective probabilities. It's aim is to aid with optimal charge selection.

- Stripping Medium

The stripping medium can be either gas or carbon foil. The current version of the program is designed to make calculations under the assumption that either Carbon foils are used (foil stripper) or Nitrogen (gas stripper).

- Input Parameters

The  $Z$  (ion atomic number),  $m$  (mass in amu),  $E$  (beam energy in MeV) and the initial ionization parameters of the beam, as well as a multiplication factor.

- Predictions

The predictions are made with use of empirical formulas that follow the assumption that the charge state distribution can be approximated with a Gaussian distribution. The formula aim to predict the probable charge state  $q_0$  produced and the width  $d$  of the charge state distribution after stripping.

- ❖ This work was conducted within the APAPES (Atomic Physics with Accelerators: Projectile Electron Spectroscopy) initiative.

# Formulas

The formulas used to calculate the mean charge state and the width of the charge state distribution are mentioned below:

- Nikolaev - Dmitriev formula (Foil stripping) - one of the best formulas for Carbon stripper foils, medium/high Z and few MeV/A
- Sayer's formula (Gas and Foil stripping) - appropriate for heavier elements
- Betz's formula (Foil stripping) - one of the best formulas for Carbon stripper foils, medium/high Z and few MeV/A
- Schiwietz - Schmitt (Gas and Foil stripping) - appropriate for elements between He - C.

For the calculation of the  $q_{\text{mean}}$  value Schiwietz's formulas for gas and foil stripping are used, while for the width calculation Schmitt's formula is used. Presented only for  $2 < z < 6$  values, where it derives best results.

# Acknowledgments

This research has been co-financed by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) Research Funding Program: THALES. Investing in knowledge society through the European Social Fund, grant number MIS 377289.

# References

- Based on "Charge", program by Justin M. Sanders [Fortran] .
- Formulas from:
  - ❖ V. S. Nikolaev and I. S. Dmitriev, 1968, "On the equilibrium charge distribution in heavy element ion beams", Physics Letters, vol. 28A, pp. 277-278
  - ❖ H. D. Betz, 1983, "Heavy- ion charge states", Academic Press, Applied Atomic Collision Physics (S. Datz, ed.), p. 1
  - ❖ RO Sayer, 1977, "Semi-empirical formulas for heavy-ion stripping data", Rev. de Phys. App. 12 (1543)
  - ❖ G. Schiwietz and P. L. Grande, 2001, "Improved Charge - State Formulas", Elsevier, Nuclear Instruments and Methods in Physics Research B 175 - 177, 125 - 131
  - ❖ C. J. Schmitt, 2010, "Equilibrium charge state distribution of low- z ions incident on thin self- supporting foils", [PhD] Dissertation, Notre Dame, Indiana.

# Interface

Calculating Charge States

Data Input

Z values Z:  [Mass values](#) Mass [amu]:  Energy [MeV]:

Incoming Charge:  [Default: 1] Factor:  [Default: 1] u [mm/ns]:

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schwietz - Schmitt formula]	1/10 [Foil Stripper, Schwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schwietz - Schmitt Formula]	1/10 [Gas Stripper, Schwietz - Schmitt formula]

Calculating Charge States

Data Input

[Z values](#) Z :  [Mass values](#) Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/ns] : 0.0

Calculate About

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schwietz - Schmitt formula]	1/10 [Foil Stripper, Schwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schwietz - Schmitt Formula]	1/10 [Gas Stripper, Schwietz - Schmitt formula]

Data Input

[Z values](#) Z :  [Mass values](#) Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/ns] : 0.0

Export to Excel

Export to png



Calculating Charge States

Data Input

Z values Z :  Mass values Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/ns] : 0.0

Calculate About

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schwietz - Schmitt formula]	1/10 [Foil Stripper, Schwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schwietz - Schmitt Formula]	1/10 [Gas Stripper, Schwietz - Schmitt formula]
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Data Input

Z values Z :  Mass values Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/ns] : 0.0

Export to Excel

Links to web pages with information about the z and mass values of the various elements

Calculating Charge States

Data Input

Z values Z :  [Mass values](#) Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/ns] : 0.0

Calculate

About

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schwietz - Schmitt formula]	1/10 [Foil Stripper, Schwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schwietz - Schmitt Formula]	1/10 [Gas Stripper, Schwietz - Schmitt formula]

Calculate

Export to Excel

Export to png

Calculating Charge States

Data Input

Z values Z:  Mass values Mass [amu]:  Energy [MeV]:

Calculate

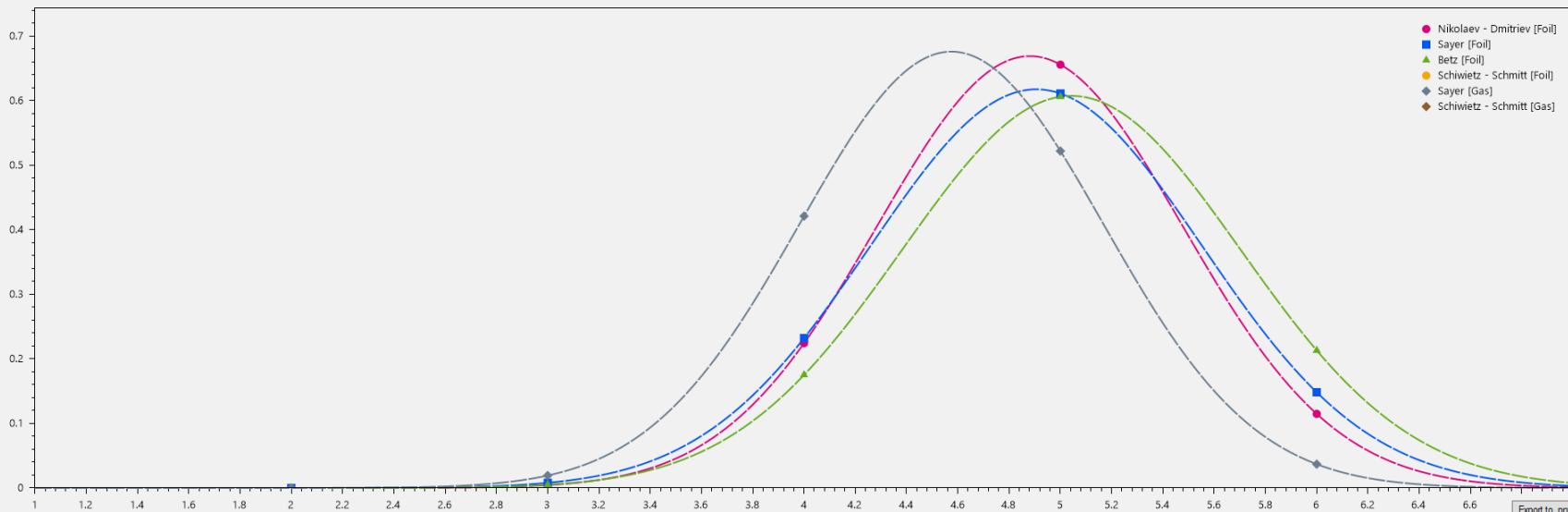
About

Incoming Charge:  [Default: 1] Factor:  [Default: 1] u [mm/ns]: 13.873062

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/I/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/I/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/I/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schiwietz - Schmitt formula]	1/I/10 [Foil Stripper, Schiwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/I/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schiwietz - Schmitt Formula]	1/I/10 [Gas Stripper, Schiwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
2	5.66E-06	1.13E-05	2.62E-05	5.25E-05	1.54E-05	3.08E-05	0.164	0.329	5.13E-05	0.000103	0.165	0.33
3	0.0046	0.0138	0.00813	0.0244	0.00515	0.0155	0.167	0.502	0.0194	0.0583	0.167	0.502
4	0.225	0.898	0.232	0.929	0.175	0.701	0.169	0.676	0.421	1.69	0.169	0.675
5	0.656	3.28	0.611	3.06	0.606	3.03	0.17	0.848	0.522	2.61	0.169	0.846
6	0.115	0.689	0.148	0.89	0.213	1.28	0.169	1.02	0.037	0.222	0.168	1.01

Export to Excel

Charge States Predictions



Export to png

Calculating Charge States

Data Input

Z values Z:  Mass values Mass [amu]:  Energy [MeV]:

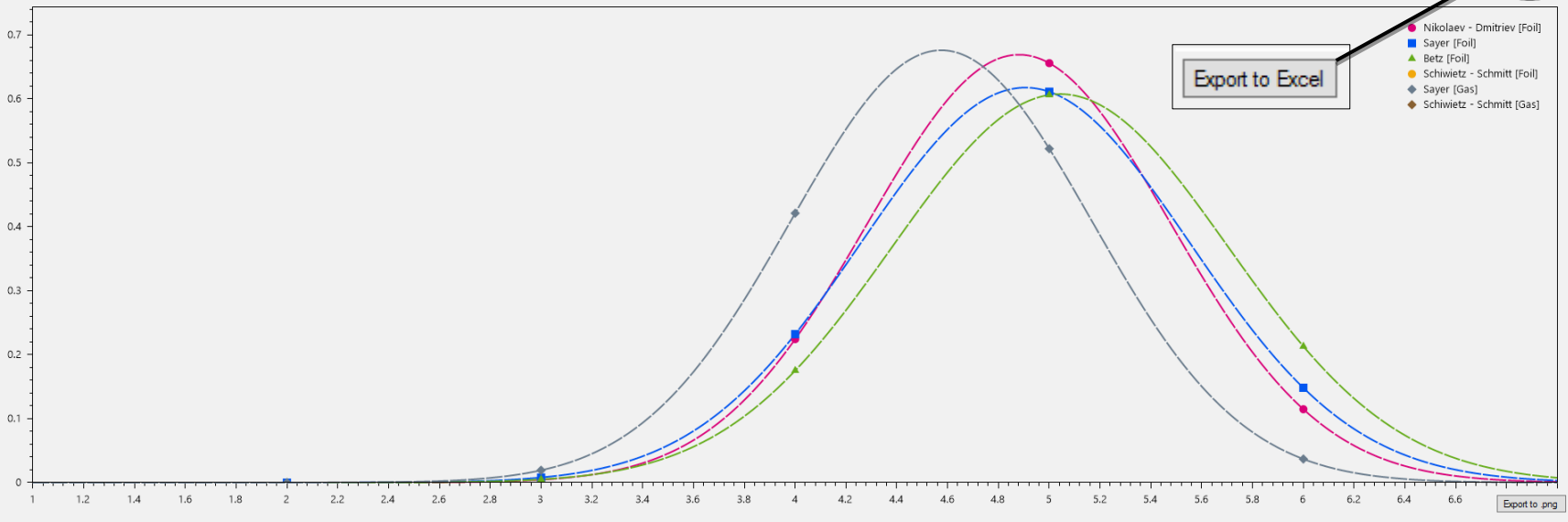
Calculate

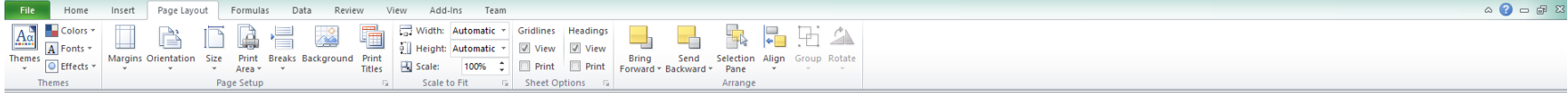
About

Incoming Charge:  [Default: 1] Factor:  [Default: 1] u [mm/ns]: 13.873062

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	I/10 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	I/10 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	I/10 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schiwietz - Schmitt formula]	I/10 [Foil Stripper, Schiwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	I/10 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schiwietz - Schmitt Formula]	I/10 [Gas Stripper, Schiwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
2	5.66E-06	1.13E-05	2.62E-05	5.25E-05	1.54E-05	3.08E-05	0.164	0.329	5.13E-05	0.000103	0.165	0.33
3	0.0046	0.0138	0.00813	0.0244	0.00515	0.0155	0.167	0.502	0.0194	0.0583	0.167	0.502
4	0.225	0.898	0.232	0.929	0.175	0.701	0.169	0.676	0.421	1.69	0.169	0.675
5	0.656	3.28	0.611	3.06	0.606	3.03	0.17	0.848	0.522	2.61	0.169	0.846
6	0.115	0.689	0.148	0.89	0.213	1.28	0.169	1.02	0.037	0.222	0.168	1.01

Charge States Predictions





	A	B	C	D	E	F	G	H
2				Foil Stripper		Gas Stripper		
3								
4								
5			Charge (q)	F(q)	I / I0			
6				[Foil Stripper, Nikolaev - Dmitriev formula]	[Foil Stripper, Nikolaev - Dmitriev formula]			
7			1	4,16E-10	4,16E-10			
8			2	5,66E-06	1,13E-05			
9			3	0,0046	0,0138			
10			4	0,225	0,898			
11			5	0,656	3,28			
12			6	0,115	0,689			
13								
14			Charge (q)	F(q)	I/I0	F(q)	I/I0	
15				[Foil Stripper, Sayer Formula]	[Foil Stripper, Sayer formula]	[Gas Stripper, Sayer Formula]	[Gas Stripper, Sayer formula]	
16			1	7,80E-09	7,80E-09	7,73E-09	7,73E-09	
17			2	2,62E-05	5,25E-05	5,13E-05	0,000103	
18			3	0,00813	0,0244	0,0194	0,0583	
19			4	0,232	0,929	0,421	1,69	
20			5	0,611	3,06	0,522	2,61	
21			6	0,148	0,89	0,037	0,222	
22								
23			Charge (q)	F(q)	I/I0			
24				[Foil Stripper, Betz Formula]	[Foil Stripper, Betz Formula]			
25			1	4,67E-09	4,67E-09			
26			2	1,54E-05	3,08E-05			
27			3	0,00515	0,0155			
28			4	0,175	0,701			
29			5	0,606	3,03			
30			6	0,213	1,28			
31								
32			Charge (q)	F(q)	I/I0	F(q)	I/I0	
33				[Foil Stripper, Schwietz - Schmitt formula]	[Foil Stripper, Schwietz - Schmitt formula]	[Gas Stripper, Schwietz - Schmitt Formula]	[Gas Stripper, Schwietz - Schmitt formula]	
34			1	0,161	0,161	0,161	0,161	
35			2	0,164	0,329	0,165	0,33	
36			3	0,167	0,502	0,167	0,502	
37			4	0,169	0,676	0,169	0,675	
38			5	0,17	0,848	0,169	0,846	
39			6	0,169	1,02	0,168	1,01	
40								
41								
42								

Calculating Charge States

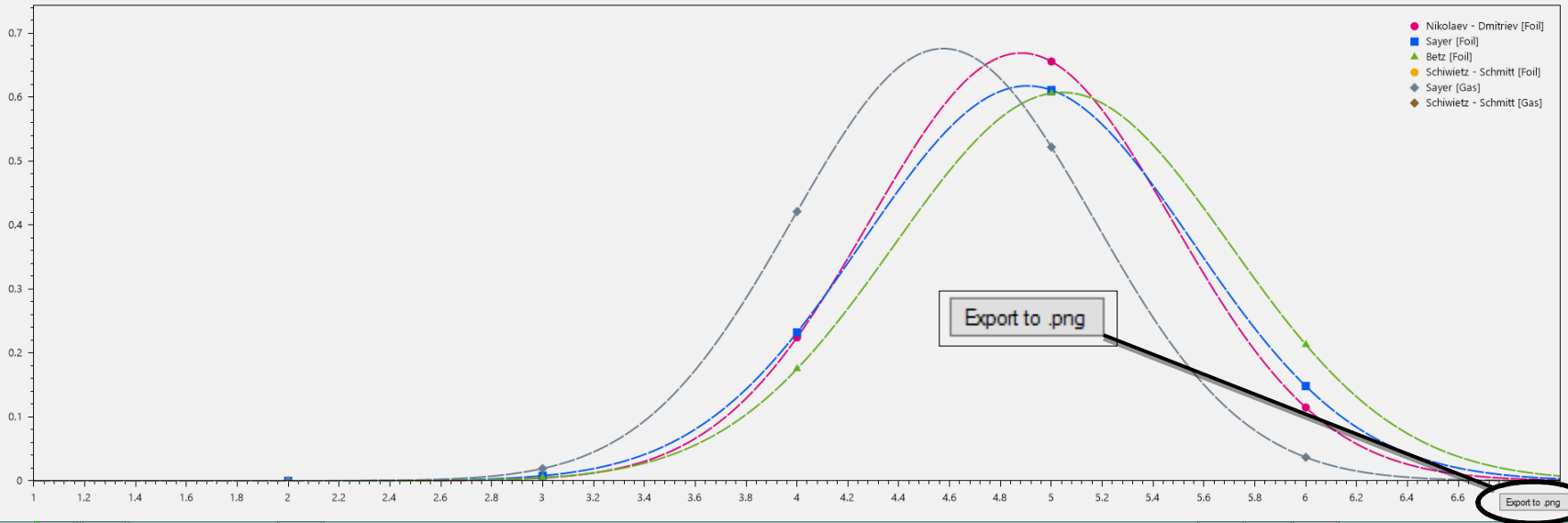
Data Input

Z values Z:  Mass values Mass [amu]:  Energy [MeV]:

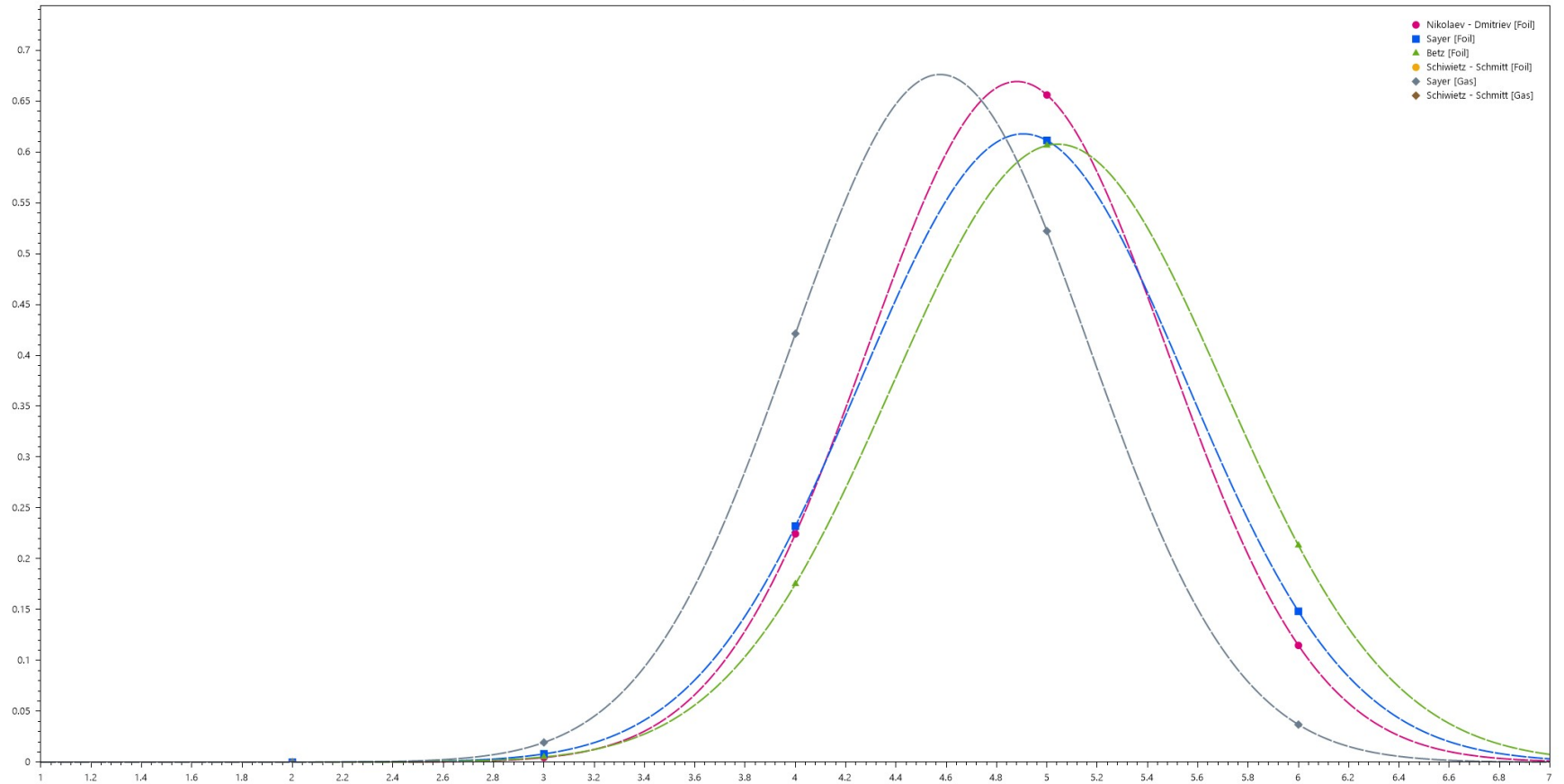
Incoming Charge:  [Default: 1] Factor:  [Default: 1] u [mm/ra]: 13.873062

Charge (e)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1 / I/O [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1 / I/O [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1 / I/O [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schiwietz - Schmitt formula]	1 / I/O [Foil Stripper, Schiwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1 / I/O [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schiwietz - Schmitt Formula]	1 / I/O [Gas Stripper, Schiwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
2	5.66E-06	1.13E-05	2.62E-05	5.25E-05	1.54E-05	3.08E-05	0.164	0.329	5.13E-05	0.000103	0.165	0.33
3	0.0046	0.0138	0.00813	0.0244	0.00515	0.0155	0.167	0.502	0.0194	0.0583	0.167	0.502
4	0.225	0.898	0.232	0.929	0.175	0.701	0.169	0.676	0.421	1.69	0.169	0.675
5	0.656	3.28	0.611	3.06	0.606	3.03	0.17	0.848	0.522	2.51	0.169	0.846
6	0.115	0.689	0.148	0.89	0.213	1.28	0.169	1.02	0.037	0.222	0.168	1.01

Charge States Predictions



### Charge States Predictions



Calculating Charge States

Data Input

Z-values Z:  Mass values Mass [amu]:  Energy [MeV]:

Calculate

About

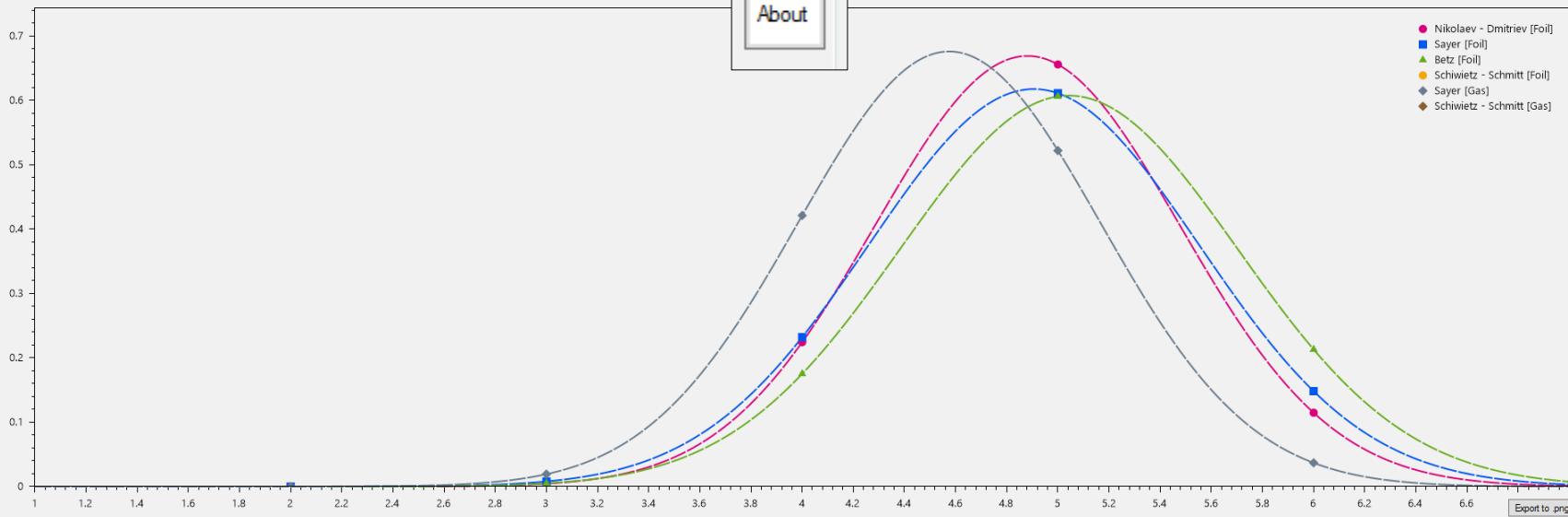
Incoming Charge:  [Default: 1] Factor:  [Default: 1] u [mm/ns]:

Charge (q)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1/I0 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1/I0 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1/I0 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schwietz - Schmitt formula]	1/I0 [Foil Stripper, Schwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1/I0 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schwietz - Schmitt Formula]	1/I0 [Gas Stripper, Schwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
2	5.66E-06	1.13E-05	2.62E-05	5.25E-05	1.54E-05	3.08E-05	0.164	0.329	5.13E-05	0.000103	0.165	0.33
3	0.0046	0.0138	0.00813	0.0244	0.00515	0.0155	0.167	0.502	0.0194	0.0583	0.167	0.502
4	0.225	0.898	0.232	0.929	0.175	0.701	0.17	0.676	0.421	1.69	0.169	0.675
5	0.656	3.28	0.611	3.06	0.606	3.03	0.17	0.848	0.522	2.61	0.169	0.846
6	0.115	0.689	0.148	0.89	0.213	1.28	0.169	1.02	0.037	0.222	0.168	1.01

Charge Distributions

Export to Excel

About



Export to png



Calculating Charge States

Data Input

Z values Z :  Mass values Mass [amu] :  Energy [MeV] :

Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/re] : 13.873062

Charge (e)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1 / I0 [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1 / I0 [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1 / I0 [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schiwietz - Schmitt formula]	1 / I0 [Foil Stripper, Schiwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1 / I0 [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schiwietz - Schmitt Formula]	1 / I0 [Gas Stripper, Schiwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
2	5.66E-06	1.12E-05	2.43E-05	5.26E-05	1.64E-05	2.08E-05	0.329	0.329	5.13E-05	0.000103	0.165	0.33
3	0.0046	0.0138					0.502	0.502	0.0194	0.0583	0.167	0.502
4	0.225	0.898					0.676	0.676	0.421	1.69	0.169	0.675
5	0.656	3.28					0.848	0.848	0.522	2.61	0.169	0.846
6	0.115	0.689					1.02	1.02	0.037	0.222	0.168	1.01

**About**

The present program is designed to calculate the expected charge states produced once an Ion beam passes through a stripping point and aid with optimal charge selection.

The stripping medium can be either gas or carbon foil.

The Z (ion atomic number), m (mass in amu), E (beam energy in MeV) and the initial ionization parameters of the beam, as well as a multiplication factor, are specified.

The predictions are made with use of empirical formulas that follow the assumption that the charge state distribution can be approximated with a Gaussian distribution.

Therefore they aim to predict the probable charge state  $q_0$  produced and the width  $d$  of the charge state distribution after stripping.

The formulas used to calculate the mean charge state and the width of the charge state distribution are the one mentioned below:

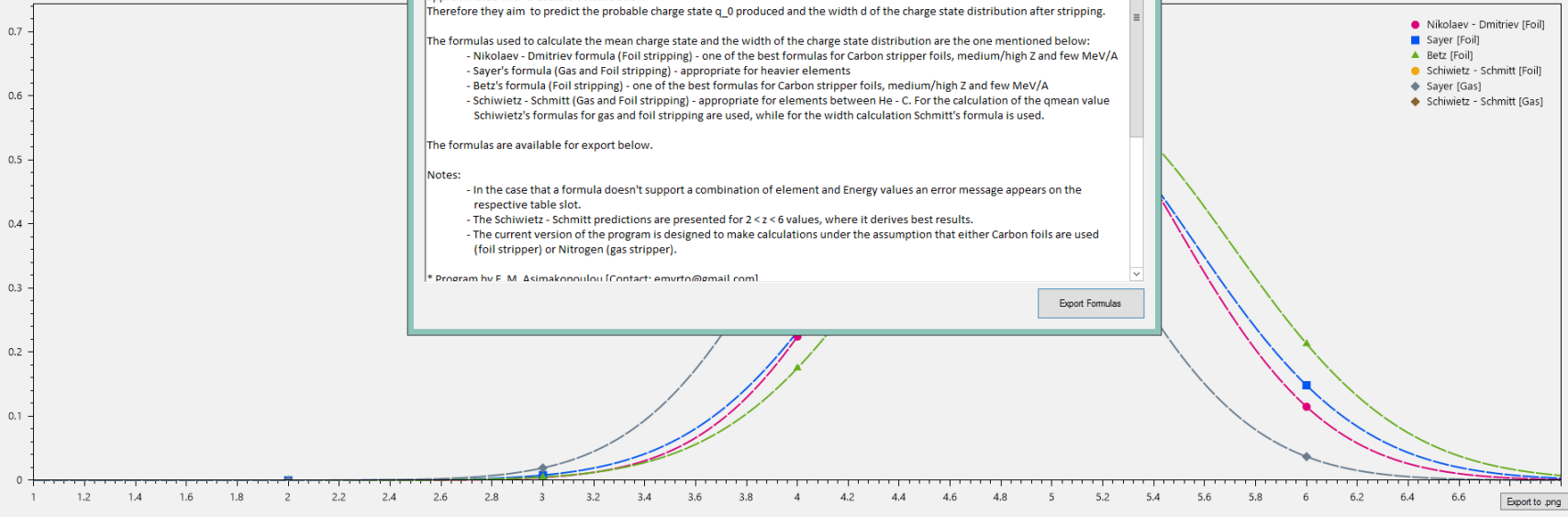
- Nikolaev - Dmitriev formula (Foil stripping) - one of the best formulas for Carbon stripper foils, medium/high Z and few MeV/A
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- Betz's formula (Foil stripping) - one of the best formulas for Carbon stripper foils, medium/high Z and few MeV/A
- Schiwietz - Schmitt (Gas and Foil stripping) - appropriate for elements between He - C. For the calculation of the qmean value Schiwietz's formulas for gas and foil stripping are used, while for the width calculation Schmitt's formula is used.

The formulas are available for export below.

**Notes:**

- In the case that a formula doesn't support a combination of element and Energy values an error message appears on the respective table slot.
- The Schiwietz - Schmitt predictions are presented for  $2 < z < 6$  values, where it derives best results.
- The current version of the program is designed to make calculations under the assumption that either Carbon foils are used (foil stripper) or Nitrogen (gas stripper).

\* Program by F. M. Asimakopoulou [Contact: amurto@gmail.com]



Calculating Charge States

Data Input

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Incoming Charge :  [Default: 1] Factor :  [Default: 1] u [mm/re] : 13.873062

Charge (e)	F(q) [Foil Stripper, Nikolaev - Dmitriev formula]	1 / I/O [Foil Stripper, Nikolaev - Dmitriev formula]	F(q) [Foil Stripper, Sayer Formula]	1 / I/O [Foil Stripper, Sayer formula]	F(q) [Foil Stripper, Betz Formula]	1 / I/O [Foil Stripper, Betz Formula]	F(q) [Foil Stripper, Schiwietz - Schmitt formula]	1 / I/O [Foil Stripper, Schiwietz - Schmitt formula]	F(q) [Gas Stripper, Sayer Formula]	1 / I/O [Gas Stripper, Sayer formula]	F(q) [Gas Stripper, Schiwietz - Schmitt Formula]	1 / I/O [Gas Stripper, Schiwietz - Schmitt formula]
1	4.16E-10	4.16E-10	7.8E-09	7.8E-09	4.67E-09	4.67E-09	0.161	0.161	7.73E-09	7.73E-09	0.161	0.161
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 The stripping medium can be either gas or carbon foil.  
 The Z (ion atomic number), m (mass in amu), E (beam energy in MeV) and the initial ionization parameters of the beam, as well as a multiplication factor, are specified.

The predictions are made with use of empirical formulas that follow the assumption that the charge state distribution can be approximated with a Gaussian distribution.  
 Therefore they aim to predict the probable charge state  $q_0$  produced and the width  $d$  of the charge state distribution after stripping.

The formulas used to calculate the mean charge state and the width of the charge state distribution are the one mentioned below:

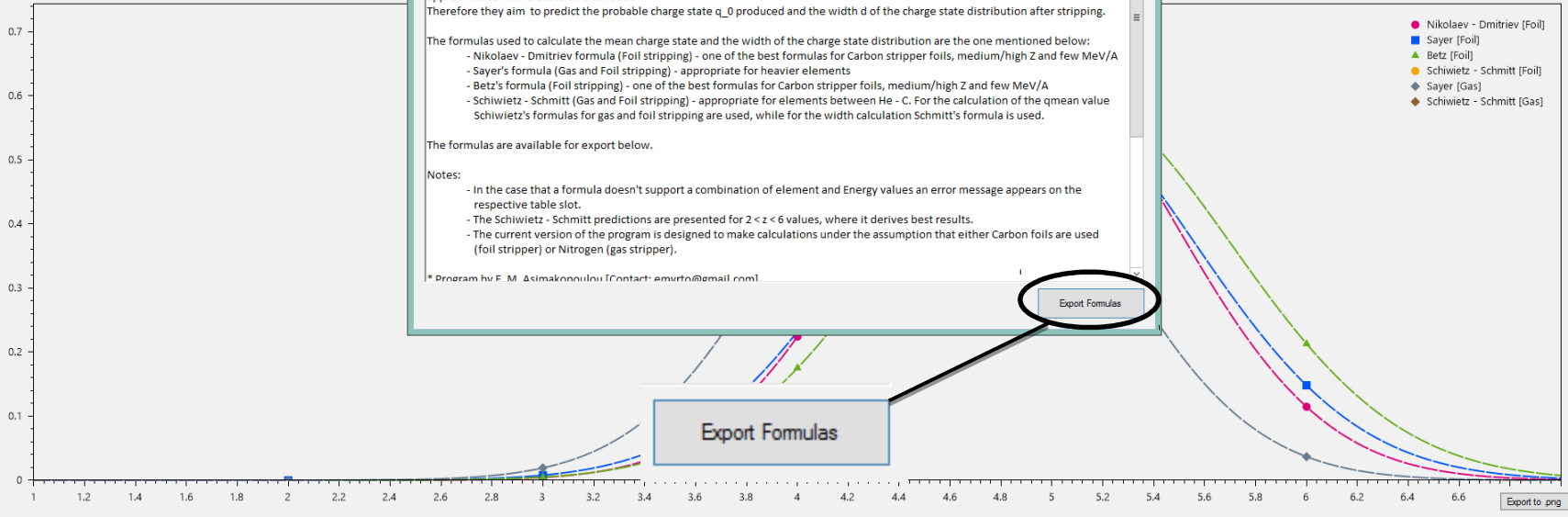
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The formulas are available for export below.

Notes:

- In the case that a formula doesn't support a combination of element and Energy values an error message appears on the respective table slot.
- The Schiwietz - Schmitt predictions are presented for  $2 < z < 6$  values, where it derives best results.
- The current version of the program is designed to make calculations under the assumption that either Carbon foils are used (foil stripper) or Nitrogen (gas stripper).

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Export Formulas

## Nikolaev - Dmitriev

### Foil Formulas

$$\bar{q} = z \left( 1 + \left( z^{-0.45} \cdot \frac{u[\frac{cm}{z}]}{3 \cdot 10^8} \right)^{-5/3} \right)^{-3/5} \quad d = \frac{1}{2} \sqrt{q_0 \left( 1 - \left( \frac{q_0}{z} \right)^{5/3} \right)}$$

### Gaussian

$$F_q = 0.398942 \cdot \frac{e^{-\frac{(q-q_{mean})^2}{d}}}{d}$$

## Betz

### Foil Formulas

$$\bar{q} = z \left( 1 - 1.041 e^{-0.851 z^{-0.432} \left( \frac{u}{u_b} \right)^{0.847}} \right) \quad d = 0.27 \sqrt{z}$$

### Gaussian

$$F_q = \frac{e^{-\frac{(q-q_{mean})^2}{2d^2}}}{d \sqrt{2\pi}}$$

## Sayer

### Foil Formulas

$$\bar{q} = z \left( 1 - e^{-55.8 z^{-0.421}} \right)^{0.2}$$

$$r = 0.38 z^{0.51} \left( \frac{\bar{q}}{z} \left( 1 - \frac{\bar{q}}{z} \right) \right)^{0.2}$$

$$ep = \begin{cases} 0 & , z \leq 15 \\ r (0.0007z - 0.7 \frac{u}{c}) & , z \geq 15 \end{cases}$$

### Gas Formulas

$$\bar{q} = \begin{cases} z \alpha \frac{u}{c} & , \frac{u}{c} \leq br \\ z \left( 1 - 1.08 e^{-80.1 z^{-0.506} \left( \frac{u}{z} \right)^{0.996}} \right) & , \frac{u}{c} \geq br \end{cases}$$

$$r = 0.35 z^{0.55} \left( \frac{\bar{q}}{z} \left( 1 - \frac{\bar{q}}{z} \right) \right)^{0.27}$$

$$ep = \begin{cases} 0 & , z \leq 15 \\ r (0.17 + 0.0012 z - 3.3 \frac{u}{c}) & , z \geq 15 \end{cases}$$

### Gaussian

$$F_q = 0.398942 \cdot \frac{e^{-\frac{1}{2} \frac{(q-q_{mean})^2}{r \left( 1 + ep \cdot \left( \frac{q-q_{mean}}{r} \right) \right)}}}{r}$$

### Note:

where  $br$ ,  $\alpha$ : breaking point and slope respectively.  
Refer to Sander's program for further info about these constants.

## Schiwietz - Schmitt

### Foil Formulas

$$\bar{q} = z \frac{12 x + x^4}{\frac{0.07}{z} + 6 + 0.3 \sqrt{x} + 10.37 x + x^4}$$

$$x = \left( z^{-0.52} \frac{u}{u_b} z_t^{-0.019 z^{-0.52} \frac{u}{u_b}} \right)$$

### Gas Formulas

$$\bar{q} = z \frac{376 x + x^6}{1428 - 1206 \sqrt{x} + 698 x + x^6}$$

$$x = \left( z^{-0.52} \frac{u}{u_b} z_t^{0.03 - 0.017 z^{-0.52} \frac{u}{u_b}} \right)^{1 + \frac{0.4}{z}}$$

### Gaussian

$$F_q = \frac{e^{-\frac{(q-q_{mean})^2}{2d^2}}}{d \sqrt{2\pi}}$$

### Note:

$$\log\left(\frac{d}{z^{1/2}}\right) = \begin{cases} -0.360 Y - 0.2397 & , \text{for } (y \geq 2 \text{ for He} - 8) \\ A + \sum_{i=2}^m B_i Y^i & , \text{and } (y \geq 2.5 \text{ for C}) \\ \text{else} & , \text{else} \end{cases}$$

$$Y = 3.86 z^{-0.45} \sqrt{\frac{E(MeV)}{m(u)}}$$

$$u_b = 2.188 \cdot 10^6 \frac{m}{s} \quad \text{Bohr's velocity}$$